Mathcad Tool for Design of Volute of Centrifugal Air Compressor

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Abstract— This paper presents a project work aimed at developing a Mathcad tool capable of designing different types of industrial centrifugal air compressor volutes. Six different volutes are designed in this project with the help of suitable algorithms, which are Rectangular Cross Sectional External and Semi-External volutes; Elliptical Cross Sectional External and Semi External volutes; Circular Cross Sectional External and Semi-External volutes. The geometry generated incorporates the manufacturing and performance requirements necessary for a practical volute design. The Mathcad tool capability is validated by comparing with the prediction of an established Commercial Tool called COMPAERO$^{©}$. Thus, this project ultimately looks in introducing a volute design tool compatible to the industry.

Index Terms— Algorithms for Mathcad, CAM design, Circular Cross Sectional External and Semi-External Volutes, Elliptical Cross Sectional External and Semi External Volutes, Rectangular Cross Sectional External and Semi-External Volutes, Validation with COMPAERO$^{©}$, Volute or Scroll

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1 INTRODUCTION

THE main objective of the project is to generate the cross sectional profiles of the volute and hence arrive at the 3D structure of the volute. The tool is developed as a Mathcad script using the commercially available design software called "Mathcad-14". Design methodology adopted is to establish the area distribution in the volute from the principles of Conservation of Angular momentum and Mass (CAM Designs) [1].
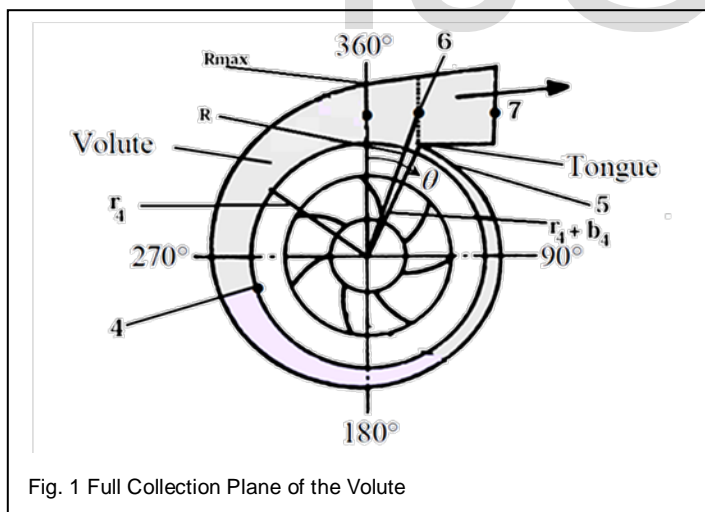

Fig. 1 Full Collection Plane of the Volute

The tool developed is capable of designing six types of commonly used industrial volutes which are as follows:-

I.   Rectangular Cross sectional External Volute

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II.   Rectangular Cross sectional Semi-External Volute
III.   Elliptical Cross sectional External Volute
IV.   Elliptical Cross sectional Semi-External Volute
V.   Circular Cross sectional External Volute
VI.   Circular Cross sectional Semi-External Volute

2 DESIGN ALGORITHMS

The algorithms used for designing the Elliptical and Circular cross sectional volutes are done in such a way that 3/4th of the volute profile is an ellipse or circle and the rest is rectangle for manufacturability. The equations used for the design can be referred in the Appendix A.

2.1 Algorithm for Rectangular External Volute
a)   Algorithm for Geometrical properties for the Full Collection Plane.
   ➢ Input Values:- Volute Inlet Radius; Volute inlet Width; Sizing parameter; Aspect Ratio; Inlet Flow angle; Number of Stations for Volute generation
   ➢ Equations for finding Variables like Height, Width, Area, Mean Radius and Maximum Radius for the Volute Passage cross section are derived from CAM designs and Geometrical concepts
   ➢ If Width ≤ Volute inlet width then, Width = Volute inlet width and continue solving the equations by keeping this value in finding other variables
   ➢ Otherwise, continue with the value obtained for Width from CAM design equations and use that to find further variables
   ➢ This process is continued for the corresponding number of Stations
   ➢ Display Values of Variables in a table format
b)   Algorithm for 3D Co-ordinates for Full Collection Plane
   ➢ Input Values:- Height, Width, Mean Radius and

Maximum Mean Radius of the Volute Passage, Number of Stations for Volute Generation
- ➢ Equations for finding the 3D Coordinates(X, Y, Z) at 5 points of the Rectangle which are Volute inlet Point; the three Corner points of the Rectangle and the Volute inlet width point derived from mathematical modeling concepts
- ➢ The process is continued for all the stations specified
- ➢ Display the coordinate values in a table format

c) Algorithm for Geometrical Properties of the Exit Cone
- ➢ Input Values:- Ideal Divergence angle needed for the Exit cone; the geometric properties of the rectangle at 360^0 of the Full Collection Plane; Number of sections for Exit cone generation
- ➢ Equations derived from basic geometrical concepts of frustum to find the height, width and area of the Exit cone Cross section at each section after giving a fixed value to the Exit Flange Area
- ➢ All the parameters of the Exit cone are calculated and displayed in the Output

d) Algorithm for Coordinates of the Exit Cone
- ➢ Input Values:- Geometrical Properties & Coordinates of the Rectangle at 360^0 of the Full Collection Plane; Geometrical Properties of the Exit Cone at each section
- ➢ Equations are derived from Mathematical Modeling concepts for finding the 3D spacing of the Exit Cone at each section
- ➢ Results are displayed in Array format showing the Coordinate values of the Exit cone

2.2 Algorithm for Rectangular Semi-External Volute

a) Algorithm for Geometrical Properties of Full Collection Plane
- ➢ Input Values:- Volute Inlet Radius; Volute inlet Width; Sizing parameter; Aspect Ratio; Inlet Flow angle; Number of Stations for Volute generation; Thickness between the annular passage and the Volute Passage
- ➢ Equations for finding the Height, Width, Area, Mean Radius and Maximum Mean Radius of the Volute Passage Derived from the CAM design and geometrical concepts
- ➢ If Aspect Ratio (Width/Height) > 1 then take the Equation of Height and check for all the stations and if Height ≤ 2*Volute inlet width then Height =2* Volute inlet width and continue solving the equations by keeping this value in finding other Variables. Otherwise continue with the value obtained from CAM design equations
- ➢ If Aspect Ratio (Width/Height) < 1 take the Equation of Width and check for all the stations and if Width ≤ 2*(Volute inlet width +Thickness) then Width = 2*(Volute inlet width +Thickness) and continue solving the equations by keeping this value in finding other Variables. Otherwise continue with the value obtained from CAM design

equations
- ➢ This process is continued for the corresponding number of Stations
- ➢ Display the Values in a table format

b) Algorithm for 3D Co-ordinates for Full Collection Plane
- ➢ Input Values:- Height, Width, Mean Radius and Maximum Mean radius of the Volute Cross sectional Passage; Number of Stations for Volute generation; Thickness between the annular passage and the Volute Passage
- ➢ Equations derived from the basic concepts of Mathematical modeling to get the values for X, Y, Z Coordinates for 6 points in the rectangle which include Volute inlet point, Volute width + thickness point, Volute inlet width point and the rest of 3 corner points of the Rectangle
- ➢ The process is continued for all the stations specified
- ➢ Display the coordinate values in a table format

c) Algorithm for Geometrical Properties of the Exit Cone
- ➢ Input Values:- Ideal Divergence angle needed for the Exit cone; the geometric properties of the rectangle at 360^0 of the Full Collection Plane; Thickness between the annular and Volute passage; Number of sections for Exit cone generation
- ➢ Equations derived from basic geometrical concepts of frustum to find the height, width and area of the Exit cone Cross section at each section providing a fixed value to the Exit Flange Area
- ➢ All the parameters of the Exit cone are calculated and displayed in the Output

d) Algorithm for Coordinates of the Exit Cone
- ➢ Input Values:- Geometrical Properties & Coordinates of the Rectangle at 360^0 of the Full Collection Plane; Geometrical Properties of the Exit Cone at each section; Thickness between the annular and Volute passage
- ➢ Equations are derived from Mathematical Modeling concepts for finding the 3D spacing of the Exit Cone at each section
- ➢ Results are displayed in Array format showing the Coordinate values of the Exit cone

2.3 Algorithm for Elliptical External Volute

a) Algorithm for Geometrical properties of Full Collection Plane
- ➢ Input Values:- Volute Inlet Radius; Volute inlet Width; Sizing parameter; Aspect Ratio; Inlet Flow angle; Number of Stations for Volute generation
- ➢ Equations for finding Variables like Total Height, Width, Area, Mean Radius and Maximum Radius for the Volute Passage are derived from CAM designs and Geometrical concepts
- ➢ If Width ≤ Volute inlet width then Width = Volute inlet width and continue solving the equations using this value to find values of other variables

- ➢ Otherwise, continue with the value obtained for Width from CAM design equations and use that to find further variables
- ➢ This process is continued for the corresponding number of Stations
- ➢ Display Values of Variables in a table format
- b) Algorithm for Co-ordinates of Full Collection Plane
  - ➢ Input Values:- Height, Width, Mean Radius and Maximum Mean radius of the Volute Cross sectional Passage; Number of Stations for Volute generation; Number of points for Ellipse Generation
  - ➢ Equations derived from the basic concepts of Mathematical modeling to get the values for X, Y, Z Coordinates for 'n+3' points in the Cross section which are Volute inlet point, first corner point of 1/4th rectangle portion, 'n' points for 3/4th Ellipse and Volute inlet width point
  - ➢ The process is continued for all the stations specified
  - ➢ Display the coordinate values in a table format

2.4 Algorithm for Elliptical Semi-External Volute

- a) Algorithm for Geometrical properties of Full Collection Plane
  - ➢ Input Values:- Volute Inlet Radius; Volute inlet Width; Sizing parameter; Aspect Ratio; Inlet Flow angle; Number of Stations for Volute generation; Thickness between the annular passage and the Volute Passage
  - ➢ Equations for finding the Height, Width, Area, Mean Radius and Maximum Mean Radius of the Volute Passage Derived from the CAM design and geometrical concepts
  - ➢ If Aspect Ratio (Width/Height) > 1 then take the Equation of Height and check for all the stations and if Height ≤ Volute inlet width then Height = Volute inlet width and continue solving the equations by keeping this value in finding other Variables. Otherwise continue with the value obtained from CAM design equations
  - ➢ If Aspect Ratio (Width/Height) < 1 take the Equation of Width and check for all the stations and if Width ≤ (Volute inlet width +Thickness) then Width = (Volute inlet width +Thickness) and continue solving the equations by keeping this value in finding other Variables. Otherwise continue with the value obtained from CAM design equations
  - ➢ This process is continued for the corresponding number of Stations
  - ➢ Display the Values in a table format
- b) Algorithm for 3D Co-ordinates for Full Collection Plane
  - ➢ Input Values:- Height, Width, Mean Radius and Maximum Mean radius of the Volute Cross sectional Passage; Number of Stations for Volute generation; Number of points for Ellipse Generation; Thickness between the annular passage and the

Volute Passage
- ➢ Equations derived from the basic concepts of Mathematical modeling to get the values for X, Y, Z Coordinates for 'n+3' points in the Cross section which include Volute inlet point, Volute width + thickness, Volute inlet width point, the rest of n points for the Ellipse and an inside corner point of the 1/4th rectangle
- ➢ The process is continued for all the stations specified
- ➢ Display the coordinate values in a table format

2.5 Algorithm for Circular External Volute

- a) Algorithm for Geometrical Properties of Full Collection Plane
  - ➢ Input Values:- Volute Inlet Radius; Volute inlet Width; Sizing parameter; Inlet Flow angle; Number of Stations for Volute generation
  - ➢ Equations for finding Variables like Total Height, Width, Area, Mean Radius and Maximum Radius for the Volute Passage are derived from CAM designs and Geometrical concepts
  - ➢ If Width ≤ Volute inlet width then Width = Volute inlet width and continue solving the equations using this value to find values of other variables
  - ➢ Otherwise, continue with the value obtained for Width from CAM design equations and use that to find further variables
  - ➢ This process is continued for the corresponding number of Stations
  - ➢ Display Values of Variables in a table format
- b) Algorithm for Co-ordinates of Full Collection Plane
  - ➢ Input Values:- Height, Width, Mean Radius and Maximum Mean radius of the Volute Cross sectional Passage; Number of Stations for Volute generation; Number of points for Circle Generation
  - ➢ Equations derived from the basic concepts of Mathematical modeling to get the values for X, Y, Z Coordinates for 'n+3' points in the Cross section which are Volute inlet point, first corner point of 1/4th of rectangle portion, 'n' points for 3/4th of Circular portion and Volute inlet width point
  - ➢ The process is continued for all the stations specified
  - ➢ Display the coordinate values in a table format

2.6 Algorithm for Circular Semi-External Volute

- a) Algorithm for Geometrical properties of Full Collection Plane
  - ➢ Input Values:- Volute Inlet Radius; Volute inlet Width; Sizing parameter; Aspect Ratio; Inlet Flow angle; Number of Stations for Volute generation; Thickness between the annular passage and the Volute Passage
  - ➢ Equations for finding the Height, Width, Area, Mean Radius and Maximum Mean Radius of the Volute Passage Derived from the CAM design and

geometrical concepts
- ➤ If Aspect Ratio (Width/Height) > 1 then take the Equation of Height and check for all the stations and if Height ≤ Volute inlet width then Height = Volute inlet width and continue solving the equations by keeping this value in finding other Variables. Otherwise continue with the value obtained from CAM design equations
- ➤ If Aspect Ratio (Width/Height) < 1 take the Equation of Width and check for all the stations and if Width ≤ (Volute inlet width +Thickness) then Width = (Volute inlet width +Thickness) and continue solving the equations by keeping this value in finding other Variables. Otherwise continue with the value obtained from CAM design equations
- ➤ This process is continued for the corresponding number of Stations
- ➤ Display the Values in a table format
  b) Algorithm for 3D Co-ordinates for Full Collection Plane
- ➤ Input Values:- Height, Width, Mean Radius and Maximum Mean radius of the Volute Cross sectional Passage; Number of Stations for Volute generation; Number of points for Circle Generation; Thickness between the annular passage and the Volute Passage
- ➤ Equations derived from the basic concepts of Mathematical modeling to get the values for X, Y, Z Coordinates for 'n+3' points in the Cross section which include Volute inlet point, Volute width + thickness, Volute inlet width point, the rest of n points for the Ellipse and an inside corner point of the 1/4th rectangle
- ➤ The process is continued for all the stations specified
- ➤ Display the coordinate values in a table format

The algorithm for the generation of Exit cone for Elliptical and circular cross sectional External as well as Semi-External volutes are not mentioned because the existing algorithm used for rectangular cross sectional External as well as Semi-External volutes are repeated over there.

3 MATHCAD OUTPUTS

The algorithms which are made above are converted into programs in Mathcad software so that a new tool could be created for designing the Volutes. The programs were created such that each algorithm produced an individual program for each volute. So, a total of 24 programs were created for the design of all the six volutes and all programs were clubbed together in another fresh Mathcad file. An additional program called Choice was added to it so that a user could choose between any of the six volutes and model the selected one. The Output of the combined programs consists of 12 parts which are:-
i.   Full Collection Plane Geometrical Properties
ii.   Coordinates for the Full Collection Plane
iii.   Exit Cone Geometrical Properties
iv.   Length of the Exit Cone

v.   Coordinates for the Exit Cone
vi.   Area Variation of Full Collection Plane
vii.   Circumferential Area Increase of Full Collection Plane
viii.   Radial Orientation of Full Collecton Plane
ix.   Variation of Full Collection plane around Diffuser
x.   3D Contour Plot of the Volute
xi.   Variation of the Exit Cone
xii.   Radial Orientation of the Exit cone

4 VALIDATION WITH COMPAERO©

In this section, the comparison results are reported for all the External volutes which were previously generated by Mathcad software. The procedure is repeated for Semi-External Volutes so, it's immaterial to report those results here. In COMPAERO© sofware the properties generated for internal volutes are basically the properties for Semi-External volutes. Thus, the results obtained in Mathcad for Semi-External volutes are directly compared with the internal volutes. The process used for comparison of geometrical properties is COMPAERO© results were subtracted from the Mathcad outputs. For matching the cross sectional shapes a separate mathcad file was created for each volute. In this program 5 angles were taken on a symmetric basis and the cross sectional shapes of the Volute were compared at each of these angles. In this section the cross sectional shape at the exit of the full collection plane of the Volute is shown.

4.1 Rectangular External Volute

Comparison of Full Collection Plane's Geometries

Avg% Area Error	Avg%R Error	Avg% Rmax Error	Avg% Wmax Error
0.4	-0.1	-0.1	-0.4

These values are procured by subtracting the COMPAERO© result values from Mathcad result values. A small difference is seen while comparing the results because COMPAERO© software has reduced the area by filleting the corner part of the rectangle with the radius having the magnitude of volute inlet width as shown in fig.2. The properties taken here for validating are the Area of the Full Collection Plane, Mean Radius Variation of the Volute Passage (R), Maximum Mean Radius Variation of the Volute Passage (Rmax), Maximum Width of the Cross Section of the Volute (Wmax).

Comparison of Cross sectional shape
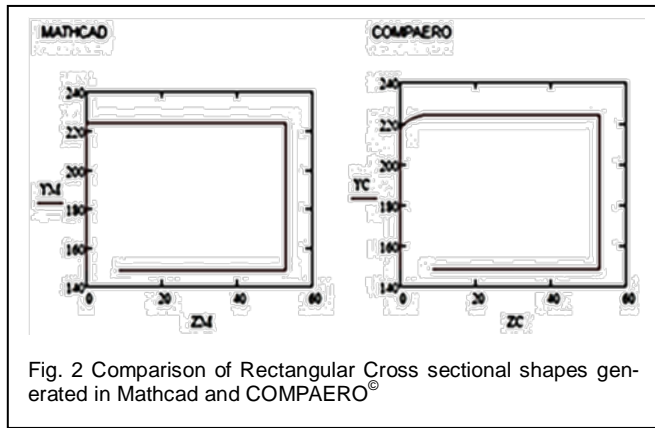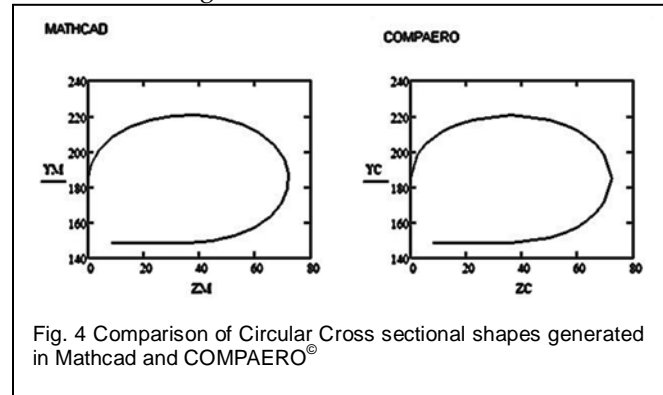
At 360^0 Polar Angle

Fig. 2 Comparison of Rectangular Cross sectional shapes generated in Mathcad and COMPAERO©

4.2 Elliptical External Volute

Comparison of Full Collection Plane's Geometries

Avg% Area Error	Avg%R Error	Avg% Rmax Error	Avg% Wmax Error
0.0	0.0	0.0	0.0

Elliptical cross sectional Volutes showed a great similarity between the results produced in both the soft wares. This tells about the authenticity of the programming algorithm used by the Mathcad software to design the Volutes.

Comparison of Cross sectional shape
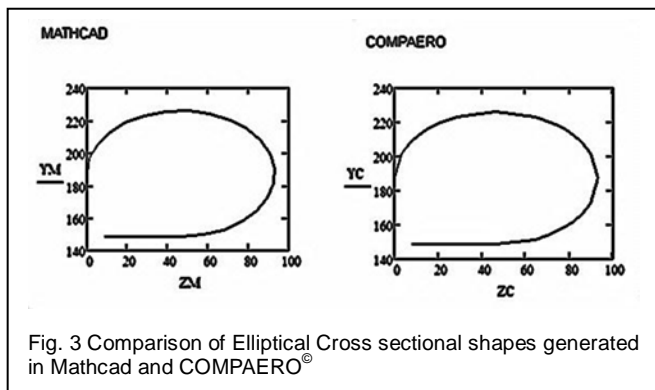
At 360^0 Polar Angle

Fig. 3 Comparison of Elliptical Cross sectional shapes generated in Mathcad and COMPAERO©

4.3 Circular External Volute

Comparison of Full Collection Plane's Geometries

Avg% Area Error	Avg%R Error	Avg% Rmax Error	Avg% Wmax Error
0.0	0.0	0.0	0.0

Circular Cross Sectional Volutes also gave a great similarity between the results obtained from COMPAERO© and from Mathcad.

Comparison of Cross sectional shape

At 360^0 Polar Angle

Fig. 4 Comparison of Circular Cross sectional shapes generated in Mathcad and COMPAERO©

5 END SECTIONS

5.1 Appendix A

Equations involved in Design of each volute are presented in this section. These equations are directly derived from the CAM design concepts of centrifugal air compressor.

For all the below volutes the equation for Area calculation is

TABLE 1
NOMENCLATURE

Variables	Representation
r	Volute Radius Measured from the center of the compressor
R	Radius of the Mean Passage flow
Rmax	Volute Outer Radius Measured from the Centre of the Compressor
b	Volute Inlet Width or Minimum width of the volute
t	Thickness between the Annular passage and the Passage flow inside the volute
H	Height of the Cross section of the volute
W	Width of the Cross section of the volute
A	Area of the Cross section of the volute
ρ	Density of the air flow in the volute
ɵ	Polar angle representing the sections of volute passage
V_f	Meridional velocity of the air
V_w	Swirl component of the velocity of the air
V	Absolute Velocity of the air
α	Flow angle with respect to the volute cross section
β	Divergence angle required for the Exit Cone
L	Length of the Exit Cone
SP	Sizing Parameter for performance
4	Volute Inlet Position
5	Full Collection Plane's maximum radial position
6	Tongue Position
7	Position of the Exit Flange Area
C	Mean Passage Flow position of the volute
E	Intermediate Exit cone position

taken from the CAM design concept [1] as,

$$(rV)_c = (rV_w)_4 \tag{1}$$

$$(\rho V)_c A = \theta(\rho r b V_f)_4 \tag{2}$$

$$A = SP * \theta * r_c * b_4 * \tan\alpha_4 \tag{3}$$

Rectangular Cross Sectional External Volute

For Rectangular Cross sectional External Volute,

$$W = \frac{1}{2}\left(\frac{S*\theta}{2} + \sqrt{\left(\frac{S*\theta}{2}\right)^2 + 4 * AR * r_4 * S\theta}\right) \tag{4}$$

Where $S = SP * b_4 * \tan\alpha_4$

Rectangular Cross sectional Semi-External Volute

For Aspect Ratio Greater than 1 (AR > 1)

$$H = \frac{1}{2*AR}\left(-\frac{t}{2} + \sqrt{\frac{t^2}{4} + 4 * (S * \theta + b_4^2 * AR)}\right) \tag{5}$$

For Aspect Ratio Less than 1 (AR < 1)

$$W = \frac{1}{2}\left(\frac{b_4-t}{2} + \sqrt{\frac{(b_4-t)^2}{4} + 4 * \theta * S}\right) \tag{6}$$

Where $S = SP * AR * r_4 * b_4 * \tan\alpha_4$

Elliptical Cross sectional External Volute

For Elliptical Cross sectional External Volute,

$$W = \left(S * \frac{\theta}{1+\frac{3}{4}*\pi} + \sqrt{\left(S * \frac{\theta}{1+\frac{3}{4}*\pi}\right)^2 + 4 * AR * r_4 * S * \frac{\theta}{1+\frac{3}{4}*\pi}}\right) \tag{7}$$

Where $S = SP * b_4 * \tan\alpha_4$

Elliptical Cross sectional Semi-External volute

For Elliptical Cross sectional Semi-External Volute,
For Aspect Ratio Greater than 1 (AR > 1)

$$H = \frac{1}{(1+\frac{3}{4}\pi)*AR}\left(-t + \sqrt{t^2 + 4 * \left(1 + \frac{3}{4}\pi\right) * (S * \theta + b_4^2 * AR)}\right) \tag{8}$$

For Aspect Ratio Less than 1 (AR < 1)

$$W = \frac{1}{\left(1+\frac{3}{4}\pi\right)}\left(\frac{b_4-t}{1} + \sqrt{\frac{(b_4-t)^2}{1} + 4 * \theta * S * \left(1 + \frac{3}{4}\pi\right)}\right) \tag{9}$$

Where $S = SP * AR * r_4 * b_4 * \tan\alpha_4$

Circular Cross Sectional External Volute

For Circular Cross sectional External Volute,

$$W = \left(S * \frac{\theta}{1+\frac{3}{4}*\pi} + \sqrt{\left(S * \frac{\theta}{1+\frac{3}{4}*\pi}\right)^2 + 4 * r_4 * S * \frac{\theta}{1+\frac{3}{4}*\pi}}\right) \tag{10}$$

Where $S = SP * b_4 * \tan\alpha_4$

Circular Cross Sectional Semi-External Volute

For Circular Cross sectional Semi-External Volute,

$$W = \frac{1}{\left(1+\frac{3}{4}\pi\right)}\left(\frac{b_4-t}{1} + \sqrt{\frac{(b_4-t)^2}{1} + 4 * \theta * S * \left(1 + \frac{3}{4}\pi\right)}\right) \tag{11}$$

Where $S = SP * r_4 * b_4 * \tan\alpha_4$

For the calculation of Exit Cone cross sectional parameters the equation used is as follows,

$$W_E = 2*W_{360}*\tan(\beta/2) \tag{12}$$

Where, W_{360} is the Width of the Cross section of the full collection plane.

5.2 Acknowledgments

6 CONCLUSION

The primary project goal of development of a volute design tool using the Mathcad was successfully completed. The new tool is capable of designing centrifugal compressor volute profiles of six different types which were mentioned all along the project. The main aim was the consideration of ease of manufacturability and performance which are of prime concern for industry which was attained with the help of this tool. The main reason for this is that the output of the tool is in a format which could be easily imported to any CAD software for modeling the same and for further manufacturing process. The Mathcad results obtained are authenticated and validated with commercially approved software called COMPAERO©, which is used in most of the major centrifugal compressor production industries. All these comparison results are reported and it has been observed that the results produced were greatly matching with the results of COMPAERO© software. Performance validation module could be taken up as a future work. Thus, it can be concluded that the Mathcad tool generated volute designs could be used in the manufacturing industries for the Centrifugal compressors.

7 REFERENCES

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